DE2 Electronics 2

Tutorial Sheet 4 – Step Response and Frequency Response

(Lectures 7 - 8)

SOLUTIONS

1.* (Refer to Lecture 7, slide 3&9) You may assume that the Laplace Transform table is available to you. See Lecture 7, slide 5-7.

Laplace transform of u(t) is 1/s. Therefore the step response of the system is in s-domain is:

$$Y(s) = \frac{1}{s} \times \frac{10}{0.1s+1} = 10\left(\frac{1}{s} - \frac{1}{s+10}\right)$$

Now take the inverse Laplace transform of Y(s) to get y(t):

$$y(t) = \mathcal{L}^{-1}\left\{10\left(\frac{1}{s} - \frac{1}{s+10}\right)\right\} = 10(u(t) - e^{-10t}u(t)) = 10(1 - e^{-10t})u(t)$$

Time taken to reach 90% of final value is:

$$9 = 10(1 - e^{-10t}) \Rightarrow e^{-10t} = 0.1$$

Therefore -10t = ln 0.1, t = 0.23 sec (or 2.3 times the time constant, which is 0.1).

2. Lecture 7, slides 13 – 16.

$$H(s) = \frac{b_0}{s^2 + a_1 s + a_0} = K\frac{{\omega_0}^2}{s^2 + 2\boldsymbol{\zeta}\, \omega_0 s + {\omega_0}^2}$$

Now modify H(s) to the form shown in this equation.

$$H(s) = \frac{512}{2s^2 + 20s + 512} = \frac{1}{2} \times \frac{16^2}{s^2 + 2 \times 5s + 16^2}$$

K = 0.5, $\omega_0 = 16\frac{rad}{sec}$, $\zeta = \frac{5}{16} = 0.313$.

Since the damping factor is less than 1, the system is underdamped.

3. $H(s) = \frac{Y(s)}{X(s)} = \frac{s+5}{s^2+5s+6}$

$$(s^2 + 5s + 6)Y(s) = (s+5)X(s)$$

Therefore the system's differential equation is:

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) = \frac{dx}{dt} + 5x(t)$$

4. To find the frequency response of the system in Q3, we substitute $s = j\omega$ into H(s):

$$H(j\omega) = \frac{j\omega + 5}{-\omega^2 + 5j\omega + 6} = \frac{5 + j\omega}{(6 - \omega^2) + 5j\omega}$$